Exercise sheet N. 10
Statistical Physics

University of Heidelberg

http://www.thphys.uni-heidelberg.de/~amendola/teaching.html

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Indicate your name, your UNI-ID and your exercise group on each page of the answer sheets.

Please hand in your solution on Tuesday 14th January during the lecture.

1 Entropy of an ideal quantum gas [10pt]

Consider an ideal quantum gas with Hamiltonian $H = \sum_\nu \epsilon_\nu n_\nu$, where ϵ_ν denotes the energy and n_ν the occupancy of quantum state ν.

1. Show that the grand canonical partition sum of the ideal quantum gas can be written as

$$\ln Z_{\mathrm{G}} = \mp \sum_\nu \ln\left(1 \mp \mathrm{e}^{-\beta(\epsilon_\nu - \mu)}\right) , \tag{1}$$

   where the upper signs apply to bosons and the lower signs to fermions.

2. Calculate the average number of particles in the gas,

$$\langle N \rangle = \sum_\nu \langle n_\nu \rangle = \frac{1}{\beta}\frac{\partial}{\partial \mu} \ln Z_{\mathrm{G}} \tag{2}$$

   for bosons and for fermions and use the result to deduce the average occupancies $\langle n_\nu \rangle$ of the quantum states.

3. Calculate the entropy

$$S = -\frac{\partial \Psi}{\partial T} = k_{\mathrm{B}}\left(1 - \beta\frac{\partial}{\partial \beta}\right) \ln Z_{\mathrm{G}} \tag{3}$$

   for bosons and fermions. Derive an expression for S, which only contains the average occupancies $\langle n_\nu \rangle$. Which result for S is obtained in the classical limit $\langle n_\nu \rangle \ll 1$?

2 Particle number fluctuations in ideal quantum fluids [15]

Calculate the variance $\sigma_N^2 = \langle N^2 \rangle - \langle N \rangle^2$ of the number of particles for

1. an ideal Bose fluid above the critical temperature $T > T_c$.
2. an ideal Fermi fluid for temperatures near $T = 0$.

Discuss the different scaling of σ_N^2 with N for Bosons and Fermions.

3 Two-dimensional ideal Bose fluid [15]

Consider a two-dimensional, ideal Bose fluid and argue that there is no Bose-Einstein condensation at finite temperature.