Exercise sheet N. 8

Statistical Physics

University of Heidelberg

http://www.thphys.uni-heidelberg.de/~amendola/teaching.html

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Indicate your name, your UNI-ID and your exercise group on each page of the answer sheets.

Please hand in your solution on Tuesday 17th December during the lecture.

1 Permanent Dipoles[10pt]

Consider a system of N permanent electric dipoles each of them with Hamiltonian $H_i = -\vec{E} \cdot \vec{p}_i$, where $\vec{p}$ is the permanent electric dipole moment of each dipole, which has a fixed modulus $|\vec{p}| = p$, and $\vec{E}$ an external electric field. The dipoles have a fixed position so that the total Hamiltonian is given by $\mathcal{H} = \sum_i H_i$.

1. Compute the partition function of the system and the Gibbs free energy $\Phi(\vec{E})$.
2. Compute the explicit expression for the polarization of the system $\vec{P} = \frac{1}{V}\sum_i \langle \vec{p}_i \rangle$.
3. Compute the susceptibility (i.e. the dielectric constant matrix $\varepsilon_{\mu\nu}$) of the system with respect to the electric field $\vec{E}$

2 Ideal Gas of particles with internal degree of freedom[10pt]

Consider an ideal gas of particles at temperature T and in a finite three dimensional box of volume V. Each particle has an internal degree of freedom which is described by a classical harmonic oscillator. The Hamiltonian of each particle reads $H_i = \frac{p_i^2}{2m} + \frac{p_s^2}{2m} + \frac{m\omega^2}{2}s^2$, where p_i is the particle momentum, s its internal coordinate and p_s the momentum associated to the internal coordinate. The frequency of the harmonic oscillator degree of freedom depends on the total volume of the system according to the law $\frac{d\log(\omega)}{d\log(V)} = \gamma$.

1. Obtain the total pressure of the system.
2. Obtain the specific heat at constant pressure.

3 Specific heat of diatomic gas[10pt]

Consider a diatomic gas of molecules composed of two atoms bound to each other along the atomic bound direction r. Taking into account also the transitional and rotational degrees of freedom of the molecules compute the specific heat of the gas in the two cases in which the atomic bound will be described by

1. a rigid rod with fixes the bound length to $r = r_0$,
2. an harmonic spring along the bound direction.

4 Specific heat of ring molecule[10pt]

Consider a single molecule formed by several atoms disposed along a ring. The molecule is free to move (and rotate) in three dimensions. Also, the molecule can pucker (i.e. create wrinkles) across its equilibrium regular ring configuration. The energy of the puckering motion can be efficiently described by the Hamiltonian of a quartic oscillator

$$H = \frac{p_s^2}{2m} + \frac{\kappa}{4} s^4, \tag{1}$$

where s is the displacement from the equilibrium ring configuration and p_s the associate momentum. Compute the specific heat of the molecule.
Hint: Use the virial theorem.