Exercise sheet N. 5
Statistical Physics

University of Heidelberg

http://www.thphys.uni-heidelberg.de/~amendola/teaching.html

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Indicate your name, your UNI-ID and your exercise group on each page of the answer sheets.

Please hand in your solution on Tuesday 26th November during the lecture.

1 The Maxwell Distribution [10pt]

The probability for a particle in a three dimensional ideal gas to have a velocity $\vec{v} \equiv (v_x, v_y, v_z)$ is given by the maxwell distribution

$$P(\vec{v}) = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv^2}{2kT}} \tag{1}$$

which only depends on the velocity modulus $v^2 \equiv v_x^2 + v_y^2 + v_z^2$. Given that the kinetic energy of each particle is $\varepsilon_{\text{kin}} = mv^2/2$, compute

1. the averege kinetic energy $\langle \varepsilon_{\text{kin}} \rangle$,
2. its standard deviation $\langle \varepsilon_{\text{kin}}^2 \rangle$,
3. the fraction of particles with $\varepsilon_{\text{kin}} > 5kT$

2 From the micro-canonical to the canonical ensemble [15pt]

1. Please describe in words the conceptual differences between the canonical and the micro-canonical ensembles, evidencing the different natural variables which characterise each ensemble.
2. Using the free energy definition $F = U - TS$ derive the relation between free-energy and partition function $Z = \int d\vec{q}d\vec{p}\exp(\beta H)/(N!h^{3N})$
3. Prove that the canonical and micro-canonical ensembles become equivalent in the thermodynamic limit. In other words, prove that the statistical fluctuations of the energy in the canonical ensemble become negligible in the thermodynamic limit. *Hint: use the fact that the specific heat is extensive $C_V \propto N$.*

3 A gas of harmonic dimers [15pt]

Consider a gas of N dimers. Each dimer is formed by two particles of equal masses m connected by a spring with elastic constant $m\omega^2$. The Hamiltonian of the system reads

$$\mathcal{H} = \sum_{i}^{2N} \frac{\vec{p}_i^2}{2m} + \sum_{i=1}^{N} \frac{1}{2} m\omega^2 |\vec{q}_{2i-1} - \vec{q}_{2i}|^2, \tag{2}$$

where $\vec{p}_i$ and $\vec{q}_i$ denote the momentum and position of the $i-th$ particle in the three dimensional space. Compute the Helmholtz free energy $F(T,V,N)$ and use it to derive:

1. The internale energy of the system.
2. The equation of state.
3. Calculate the average square length of the dimer $\langle|\vec{q}_{2i-1} - \vec{q}_{2i}|^2\rangle$.