Exercise sheet N. 4
Statistical Physics

University of Heidelberg

http://www.thphys.uni-heidelberg.de/~amendola/teaching.html

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Indicate your name, your UNI-ID and your exercise group on each page of the answer sheets.

Please hand in your solution on Tuesday 12th November during the lecture.

1 Multivariate Distributions [5pt]

Consider two correlated random variables x_1 and x_2 distributed according to the probability density function (pdf)

$$p(x_1, x_2) = A \exp\left(-\left(\frac{x_1^2}{2\sigma_1^2} + \frac{x_2^2}{2\sigma_2^2} - \frac{\rho\, x_1 x_2}{\sigma_1 \sigma_2}\right) \Big/ (1-\rho^2)\right). \tag{1}$$

Compute the normalization factor A and the covariant matrix $C_{ij} = \langle x_i x_j \rangle$, where $\langle \cdots \rangle$ defines the average over the pdf.

2 Two level systems [10pt]

Consider a system of N two states components. Each component can occupy either of two states with energy $\pm E_0$ (with $E_0 > 0$). Compute the entropy of the system as a function of the internal energy and derive the temperature of the system. What is the condition to have $T > 0$? What is the meaning of states with $T < 0$? Derive the relation between energy and temperature in the system.

3 Distinguishable particles [10pt]

Consider a system a box of volume V separated into two compartments of volumes V_1 and V_2 by a rigid membrane. The two partitions with volumes V_1 and V_2 are occupied by two different type of particles with masses m_1 and m_2. Each box contains the same density of particles $N_1/V_1 = N_2/V_2 = n$ at the same temperature $T_1 = T_2 = T$.

- Compute the entropy for the system in the assumption of distinguishable particles.
- Compute the change in entropy between the initial state and the equilibrium state obtained after the removal of the membrane.

4 Indistinguishable particles [5pt]

Consider the same system described in Ex. 3, but in this case the two volumes V_1 and V_2 are occupied by indistinguishable particles of equal mass. Prove that the change in entropy due to the removal of the barrier is zero in this case. Can you apply the same formula as in Ex. 3.

5 Statistical mechanics description of an ideal gas [10pt]

Consider a system of N free particles confined in a square box of volume L and described by the Hamiltonian

$$\mathcal{H} = \sum_{i=1}^{N} \frac{p_i^2}{2m} \tag{2}$$

where m is the particle mass and p its momentum. Assuming that the interaction between the particles can be neglected calculate the entropy of the system in the micro-canonical ensamble and prove that it reproduces the Sackur-Tetrode formula. In addition, you should

- Prove that the system follows the equation of state of the ideal gas.
- Calculate the system internal energy.
- Calculate the chemical potential of the system.
- Calculate the Helmholtz free energy of the system.