Exercise sheet N. 2

Statistical Physics

University of Heidelberg

http://www.thphys.uni-heidelberg.de/~amendola/teaching.html

Nicolò Defenu

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Indicate your name, your UNI-ID and your exercise group on each page of the answer sheets.

Please hand in your solution on Tuesday 5th November during the lecture.

1 Generalised equation of state [5pt]

Given three state variables x, y, z fulfilling the equation of state

$$F(x,y,z) = 0 \tag{1}$$

with F any three dimensional function prove that

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1 \tag{2}$$

$$\left(\frac{\partial F}{\partial x}\right)_z = \left(\frac{\partial F}{\partial x}\right)_{z,y} + \left(\frac{\partial F}{\partial y}\right)_{z,x} \left(\frac{\partial y}{\partial x}\right)_z \tag{3}$$

2 Diatomic gas [10pt]

A compressor performs adiabatic compressions on two ideal gases, first a monoatomic one (i.e. one with 3 d.o.f., translations, for instance helium) and then a diatomic one (with 5 d.o.f, 3 translations plus 2 rotational d.o.f., for instance air, to some approximation), starting with the same pressure and volume and compressing to the same final volume.

1. Show $\gamma = \frac{C_p}{C_V} = \frac{2}{d} + 1$ for an ideal gas with d d.o.f.
2. In which case does the compressor heat more?

Hint: Use the relation $C_p - C_V = T\left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_p$

3 Thermodynamic Transformation [10pt]

A volume V of ideal gas at pressure p is compressed isothermally to a volume r times smaller and then expanded adiabatically to the original volume, as in Fig. 1. What is the final pressure? What is the work done on or by the system in the two cases in which the gas is monoatomic or diatomic, as a function of the initial pressure and volume and of the compression ratio?

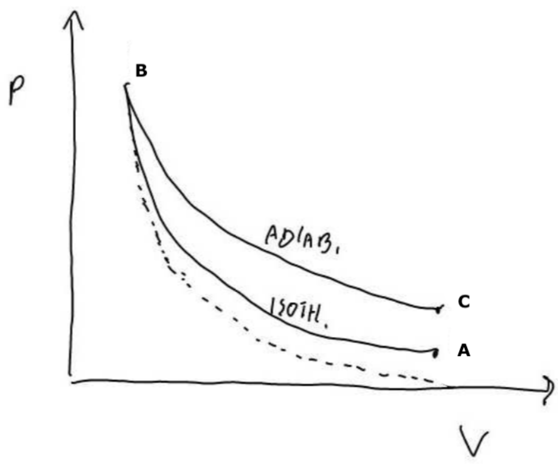

Figure 1: Isothermal compression and adiabatic expansion.

4 Speed of sound [5pt]

The sound speed of longitudinal waves of small amplitude in an ideal gas with pressure p and density ρ is

$$v = \sqrt{\frac{dp}{d\rho}} \tag{4}$$

Find the sound speed in the case in which one can assume that the rarefactions and compressions are a) adiabatic and b) isothermal.

5 Thermodynamic machine [10pt]

The following processes are performed with an ideal gas

- $1 \to 2$ adiabatic compression $\delta Q = 0$ from volume V_1 to V_2,
- $2 \to 3$ isochoric compression (i.e. increase of pressure),
- $3 \to 4$ adiabatic expansion from V_2 to V_1,
- $4 \to 1$ isochoric decrease of pressure.

Draw the $p - V$ diagram and calculate the efficiency of the loop process as functions of V_1 and V_2.
Hint: The equation of state for an adiabatic process is $pV^\gamma = K$, where K is a constant (i.e. it does not depends on the state variables) and $\gamma = 5/3$ for mono-atomic gases.