Exercise sheet N. 1
Statistical Physics

University of Heidelberg

http://www.thphys.uni-heidelberg.de/~amendola/teaching.html

October 22, 2019

Indicate your name, your UNI-ID and your exercise group on each page of the answer sheets.

Please hand in your solution on Tuesday 29th October during the lecture.

1 Playing with coins and dices (5pt)

In the two situations listed in the following identify the probability space (sample space) Ω and then calculate $A\backslash B$, i.e. the set difference $(A\,\text{or}\,B)$, which is the set of all members of A that are not members of B, and $(A\cup B)^c$, i.e. the complementary ensemble of $A\cup B$.
Hint: The intersection set $A\cup B$ is the set of all objects that are a member of A, or B, or both, while the complement set of any given set X is give by the set difference between the set X and the probability space of the problem $X^c = \Omega\backslash X$.

1. A coin is flipped three times with results "T"=tails or "H"=heads.

   A := An even number of tosses is heads,

   B := All tosses are equal.

2. A fair dice is rolled two times independently.

   A := Exact one roll is divisible by 2 and the other is strictly bigger,

   B := The sum equals 5.

2 Stirling Cycle (10pt)

Consider a monoatomic ideal gas in a reversible cycle which brings the gas in contact with two heat baths at $T_h > T_\ell$, as in Fig. (1). The isothermal processes are connected by isochores. Find the total work done by the system and the efficiency of the cycle.

3 Ideal gas in a box (10pt)

Assume an isolated box of volume V with a wall inside, that separates the inner volume V into two distinct volumes V_1 and V_2. While the space of V_2 is empty, the space of V_1 is filled with an ideal gas with N atoms. The energy of the gas is determined by the caloric equation of state E = 3/2 p_1V_1 with the pressure p_1. Suddenly the wall is removed. If the temperature of the ideal gas was T_1 before the wall was opened, which temperature will be measured in the whole box after the wall was removed? The system is depicted in Fig. 2 Calculate the change in entropy of the system before and after removing the wall.
Hint: The entropy is defined as $dS := \delta Q/T$ and follows the 1st law of thermodynamics $dS = \frac{dU+pdV}{T}$.

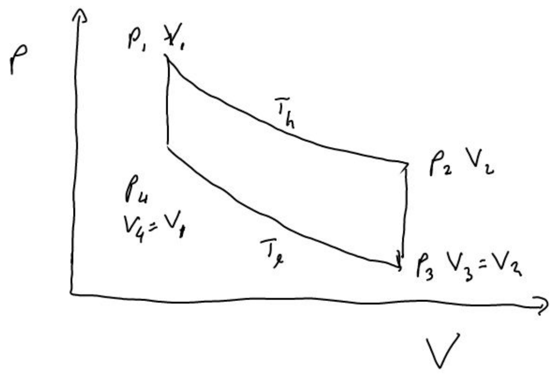

Figure 1: Find the work and efficiency of a cycle.

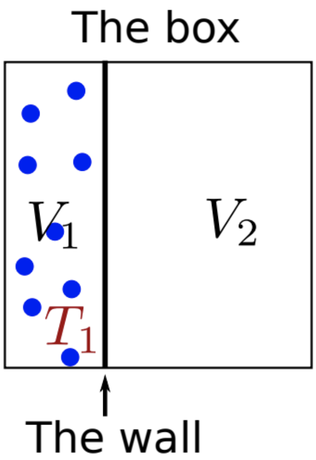

Figure 2: Schematic representation of the system.

4 Maxwell Distribution (5pt)

Given the Maxwell distribution of speed,

$$f(|v|) = \left(\frac{m}{2\pi kT}\right)^{d/2} e^{-\frac{mv^2}{2kT}} \tag{1}$$

where v^2 is the square modulus of the particle velocity and d the dimension of the system. Show that each degree of freedom contributes equally to the energy and find the root mean square velocity $\sqrt{\langle v^2 \rangle}$ as a function of the dimension d.

5 Differentials (5)

Which of these are exact differentials?

1. $\mathrm{d}f = 2xy^3\mathrm{d}x + 3x^2y^2\mathrm{d}y$
2. $\mathrm{d}f = 2x^2y^4\mathrm{d}x + 3x^3y^3\mathrm{d}y$